Leveraging the Trade-off Between Spatial Reuse and Channel Contention in Wireless Mesh Networks

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***Abstract*—Performance optimization strategies in wireless mesh networks have witnessed many diverged directions, out of which the recent studies have explored the use of spatial multiplexing through data rate and transmit power adaptation to increase spatial re-usability while minimizing network interference. However, joint data rate and transmit power adaptation shows a clear trade-off, where higher transmit power helps in sustaining high achievable data rates with the cost of increased interference to the neighboring receiver nodes. Such a trade-off results in channel access unfairness among contending flows, where a node with high transmit power gains more performance benefit compared to its neighbors. Unfairness among nodes in a mesh network results in performance drops for the end-to-end flows. In this paper, we formulate a multivariate optimization problem to maximize network utilization and fairness, while minimizing average transmit power for all transmitter nodes. The Pareto optimality nature of the vector optimization has been explored to design a distributed localized heuristic where every node individually decides their scheduling slots and transmit power while keeping fairness as a constraint. The performance of the proposed scheme has been evaluated through simulation, and the comparisons with recent studies reveal that it improves fairness that results approximately $10\% - 40\%$ improvement in end-to-end throughput for different network and traffic scenarios.**

***Keywords*-Fairness; Power adaptation; Rate Control; Scheduling; Joint Optimization; MCCA**

I. INTRODUCTION

Wireless mesh network (WMN) is one of the promising technologies for providing cost efficient and robust backbone infrastructure for broadband connection over metropolitan area networks. IEEE 802.11 task group for wireless local area network provides the standard IEEE 802.11s [1], that defines the media access control (MAC) layer channel access and forwarding for multi-hop wireless mesh networks. IEEE 802.11s supports mesh coordination function (MCF) for supporting quality of service (QoS) provisioning in mesh functionalities, which defines MCF controlled channel access (MCCA) for MAC layer scheduling of mesh nodes.

The design and performance of MAC layer protocols for a multi-hop mesh networks significantly depend on several physical layer parameters - like modulation and coding schemes (MCS), transmit powers, channel widths etc, which are further interdependent. Higher MCS levels provide better physical data rate, however require higher transmit power for sustainability. If a transmitter uses higher transmit power level, the data decoding probability at the receiver increases by supporting higher signal to interference and noise ratio (SINR). This can be realized by the following relation between transmit power, physical data rate and SINR [2];

$$SINR = \frac{P_{ij}G_{ij}}{\eta + \sum_{\substack{k\in I \\ k\neq i \\ x\neq j}} G_{kj}P_{kx}} \geq \gamma(r_h) \qquad (1)$$

Here P_{ij} and G_{ij} denote the transmit power level and "antenna and channel gain" for a wireless link between transmitter i to receiver j, respectively. I is the set of nodes in the network. η is the ambient noise and $\gamma(r_h)$ is the receive sensitivity for data rate r_h, where r_h corresponds to the physical data rate for MCS level h. If the transmission power is increased, the other receiver nodes in the communication range of the transmitter experience interference, that reduces spatial re-usability of the available channel space. From Eq. (1), it is also evident that the physical data rate has also a direct relationship with the transmit power in forms of receive sensitivity. Receive sensitivity can be defined as the required SINR for correct decoding of a frame transmitted in a particular data rate, that withstands a standard bit error rate (BER) [3]. If SINR of a received frame is greater than the receive sensitivity, then only a node can decode the frame successfully. It has been observed that higher the data rate, greater the receive sensitivity. Notationally for different data rates $r_1 < r_2 < ... < r_m$, $\gamma(r_1) < \gamma(r_2) < ... < \gamma(r_m)$. In summary, the trade-off between data rate and transmit power with respect to spatial re-usability and interference can be captured as follows:

(i) Higher data rates require higher receiver sensitivity, which implies that they can sustain only at higher SINR levels.
(ii) Higher transmit powers provide higher SINR, however in the cost of increased interference and reduced spatial re-usability of the channel.

Therefore, choosing a proper scheduling of transmit power level along with proper MCS level to gain the optimum throughput is a challenge in case of WMN. In a recent study, Li *et.al.* [4] have shown that optimal power control strategy with scheduling even for only two discrete power levels is $\mathcal{NP}$-hard because of the additive/cumulative interference effect on the receiver node.

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Optimal power control can indirectly improve system performance in terms of throughput. In their work, Cui *et.al.* [5] have suggested use of dynamic programming based solution for controlling radio parameters. Though improvement in network throughput is desirable, it does not ensure improvement in end user throughput. Therefore, increase in fairness at network level is also important as the end users experience better QoS. In this paper, we design a vector optimization problem to simultaneously control data rate, transmit power and MAC layer access parameters, that improves network fairness, spatial re-usability and end user performance. The major contributions of this paper can be summarized as follows.

1) A multi-objective optimization problem is formulated for maximizing network utilization while ensuring fairness. Simultaneously, the optimization targets in minimizing the total transmit power among the contending nodes. The solution of this problem results the assignment of transmit power and MCS along with scheduling parameters such that all links get fair share of the throughput (§III). Mathematical analysis reveals that a Pareto optimal solution for the proposed optimization problem exists (§IV).
2) The problem is centralized in nature and finding a Pareto optimal solution is $\mathcal{NP}$-hard. Therefore a distributed heuristic is proposed by exploring the properties of the centralized problem. Suitable adjustments are made for augmenting the standard MCCA for providing fairness based on the proposed mechanism (§V).
3) Extensive simulations are performed to compare the performance with that of standard MCCA and existing "Distributive Power control Rate adaptation Link scheduling" (DPRL) mechanism [2] to show the improvement in fairness and end user throughput. The results reveal that the proposed scheme results in $10\% - 40\%$ improvements in end-to-end throughput compared to DPRL, in different network and traffic generation scenarios(§VI).

II. Background and Related Works

According to the IEEE 802.11s [1] standard, a WMN consists of two types of nodes called mesh stations (mesh STA) and client stations (client STA). Mesh STAs are the wireless routing entities which form a backbone infrastructure for the client STAs. Some of the mesh STAs act as gateways (mesh gate) which connect the backbone to other network segments or the Internet. Client STAs can connect to this backbone structure for accessing the Internet. A group of mesh STAs collectively form a *"Mesh Basic Service Set" (MBSS)* that uses similar mesh profile among all the STAs.

The standard for IEEE 802.11s MAC describes a set of different channel access protocols. For contention based compulsory channel access mechanism it uses *"distributed coordination function" (DCF)*. The standard also supports for optional *"point coordination function" (PCF)* which is a polling based contention free channel access mechanism. *"Mesh coordination function" (MCF)* is another optional channel access mechanism for ensuring MAC layer quality of service (QoS) for mesh networks. MCF controlled channel access (MCCA) is a *"space-time division multiple access" (STDMA)*[6] based protocol to avoid contention during channel access and to reserve channel resources based on QoS requirements. Each period for MCCA is defined as *"delivery traffic indication message" (DTIM)* interval. DTIM interval is further slotted in MCCA opportunity (MCCAOP) slots. Each reservation consists of three parameters named 1. MCCAOP offset, 2. MCCAOP duration and 3. MCCAOP periodicity. These parameters decide transmission opportunity (TXOP) of a mesh STA. According to the IEEE 802.11s terminologies, a mesh STA that wants to send data, is called a MCCAOP owner, and the corresponding receiver mesh STA is called a MCCAOP responder. The MCCAOP owner and the MCCAOP responder decide TXOPs by exchanging a number of control frames. After the reservation is successful, both the MCCAOP owner and the MCCAOP responder broadcast a MCCA advertisement frame that consists of all the tracked reservations in the sender mesh STA and another parameter, called 'Mesh Access Fraction' (MAF). MAF determines the ratio of the total reserved time by a mesh STA over the total DTIM time period. The maximum reservation period in a DTIM interval for a mesh STA is bounded by MAF limit. It can be noted that the standard scheduling mechanism does not consider physical layer parameters, like transmit power and MCS levels, during scheduling decision.

To address the interdependency between transmit power and MCS, joint design approaches are proposed in the literature. Joint design approach considers the interference relationship between links which are dependent on power level assignment of nodes. In [7], Chen *et.al* have proposed a distributed two phase approach. However their work does not consider the effect of rate adaptation and its dependency with the SINR threshold. As mentioned earlier effective decoding of a received frame depends on mainly two parameters a) SINR and b) MCS. Hedayati *et.al.*[8] modeled the centralized joint integrated power and rate control problem in case of a STDMA network using a mixed integer linear programming (MILP). They have shown that finding a Pareto optimal solution of the MILP problem is $\mathcal{NP}$-hard. They also reduced the problem to maximum independent set problem, and proposed a greedy heuristic solution to the problem. In the subsequent works [2], they proposed a distributed heuristic protocol named DPRL. DPRL protocol uses broadcasting of SINR value. The link with highest SINR margin wins, and then it performs the power and rate calculation based on the channel quality. This process iterates until all the links are scheduled. However, the method suffers from unfairness issues.

The terminology "fair" has different notions in the literature. If achieved throughput of each mesh STA is proportional to it's traffic load then it is called *proportionally fair*. In case of *max-min fair* system, the objective is to maximize the throughput of bottleneck links. It is debatable to use *max-min fairness* [9]. Also being a global property *proportional fairness* is difficult to achieve in a distributed environment. Therefore Mo *et.al.* [9] alternately proposed $(\mathfrak{P}, \alpha)$-Proportional fairness which is a generalization of *proportional fairness* as well as *max-min fairness*. According to their work let $\mathfrak{P} = \{\mathfrak{P}_{11}, \mathfrak{P}_{12}...\mathfrak{P}_{nn}\}$ and α be all positive numbers. Now a vector of rates $\mathscr{R}^*$ is said to be $(\mathfrak{P}, \alpha)$-Proportionally fair if for any feasible vector $\mathscr{R}_{n\times n}$ under the assumption that, for a n node network with priority of communication being $\mathfrak{P}_{ij}$ between i and j.

$$\sum_{ij} \mathfrak{P}_{ij} \frac{\mathscr{R}_{ij} - \mathscr{R}^*_{ij}}{\mathscr{R}^{*\alpha}_{ij}} \leq 0 \tag{2}$$

The Eq. (3) gives a utility criterion which maximizes overall network throughput under capacity constraints of each links *iff* the rate vector $\mathscr{R}^*$ is $(\mathfrak{P}, \alpha)$-Proportionally fair, as defined in Eq. (2)[1].

$$F_\alpha(\mathscr{R}) = \begin{cases} \mathfrak{P}_{ij} \log(\mathscr{R}) & \alpha = 1 \\ \mathfrak{P}_{ij} \frac{\mathscr{R}^{(1-\alpha)}}{(1-\alpha)} & \text{Otherwise} \end{cases} \tag{3}$$

When $\alpha = 1$, Eq. (3) converges to proportional fairness, and for $\alpha \to \infty$, it reduces to max-min fairness.

However, only transmit power level and MCS selection does not ensure fair allocation of the channel. For ensuring fairness of flows, an explicit strategy needs to be employed so that each mesh STA gets fair share of throughput. It has been studied in [10] that network performance in terms of throughput has a trade-off with fairness of the system. Hence it is challenging to find a strategy which provides fairness for joint power and rate scheduling (JPRS).

III. System Model

IEEE 802.11s MAC layer specification with IEEE 802.11b/g/n physical layer supports multiple frequency channels for contention free channel access mechanism. However many community WMN uses single channel for mesh backbone because static channel assignment for a particular mesh STA limits the neighbor nodes. On the other hand dynamic channel assignment is known to be $\mathcal{NP}$-hard and current heuristics available for channel assignment are costly. So changes in standard are required. The communication channel time is divided into identical channel slots (MCCAOP) of duration σ. Each mesh STA can adjust their transmit power level and MCS from a set of available options. Let power level of mesh STA i for communication $i \to j$ at time t be denoted as $P^{(t)}_{ij}$, where $i \to j$ denotes communication between source i and destination j. Let, r_h be considered as data rate corresponding to h-th MCS. Also consider each mesh STA supports m different such MCSs. R denotes the set of all available data rates. So $R = \{r_1, r_2...r_m\}$ such that $r_1 < r_2 < ... < r_m$. This paper uses the notation $i_x \underset{r_z}{\to} j_y$ for denoting flow i_x to j_y with data rate r_z. In this context r_z and $R(x, y)$ have been used interchangeably throughout this paper.

Let $X^{(T)}_{n\times n\times m}$ denotes the channel reservation matrix of 4-dimension for a network with n mesh STAs which has the following interpretation

$$X^{(t)}_{ijh} = \begin{cases} 1 & \text{If flow i} \to \text{j uses rate } r_h \text{ at time t} \\ 0 & \text{Otherwise} \end{cases}$$

Here T represents number of MCCAOP in a DTIM period. Consider, Tx_{ij} denote number of bits sent by flow $i \to j$ in each DTIM interval. Therefore

$$\begin{aligned} Tx_{ij} &= [DU_{ij} \times Period_{ij} \times r_h \times \sigma] \\ &= \sum_{t=0}^{DTIM} \sum_h (X^{(t)}_{ijh} \times r_h \times \sigma) \end{aligned}$$

where $Period_{ij}$ and DU_{ij} denote the MCCAOP periodicity and MCCAOP duration of the corresponding sub-flow respectively. The notation of Tx has been used to refer the vector $[Tx_{i_1 j_1}, Tx_{i_2 j_2}...Tx_{i_n j_n}]$

It is commonly assumed that "antenna and channel gain" is homogeneous i.e. $G_{ij} \approx G_{ji}$ [11]. The reason behind this assumption is that all wireless routers are equipped with similar type of interfaces. Let receive sensitivity of r_h is denoted by $\gamma(r_h)$.

A feasible transmission scenario $S(t) = \{i_1 \underset{R(1,1)}{\to} j_1, i_2 \underset{R(2,2)}{\to} j_2...i_k \underset{R(k,k)}{\to} j_k\}$ under power allocation vector $P(t) = \{P^{(t)}_{i_1 j_1}, P^{(t)}_{i_2 j_2}...P^{(t)}_{i_k j_k}\}$ at time t means that the SINR level in each receiver (j_x) must be over threshold $(\gamma(R(x, x)))$. Thus Eq. (1) reduces to the following.

$$\frac{G_{i_k j_k} P^{(t)}_{i_k j_k}}{\eta + \sum_{x \neq k} G_{i_x j_k} P^{(t)}_{i_x j_x}} \geq \gamma(R(k, k)) \tag{4}$$

The power assignment for different links are not comparable because the power profile of each mesh STA has to satisfy Eq. (4) individually. Hence cumulative power can not be used as an utility function. Therefore the notion of optimality for power allocation is redefined in this context.

Definition 1: A power vector $P(t)$ is defined to be Pareto optimal for a particular feasible transmission scenario $S(t)$ where any other feasible power assignment $P'(t)$ needs atleast as much power as $P(t)$ for each mesh STA in the same transmission scenario $S(t)$. Mathematically

$$\forall z \in \{1, 2...|P(t)|\} : P^{(t)}_{i_z j_z} \leq P'^{(t)}_{i_z j_z}$$

To achieve Pareto optimal power allocation, this work proposes a multi objective integer program (MOIP) which considers power and rate allocation along with scheduling strategy. However, it has been observed that the link scheduling problem does not consider fairness issues. To achieve the fair share of network throughput, this work considers $(\mathfrak{P}, \alpha)$-proportionally fair scheduling scheme. Using the fairness criteria the objective of the system modifies to achieve Pareto optimal **J**oint **P**ower and **R**ate **S**cheduling which ensures $(\mathfrak{P}, \alpha)$-proportional fairness (Fair-JPRS). The problem tries to achieve the maximum fairness using minimum transmit power level simultaneously.

[1]Here $\log(\mathscr{R}) = \sum_i \log(\mathscr{R}_i)$

IV. Joint Scheduling for Channel Access and Transmit Power Levels: A Centralized Vector Optimization Formulation

In this section, a centralized vector optimization problem is formulated to solve the above mentioned Fair-JPRS problem. The proposed vector optimization problem (VOP) has two aspects.

1) To schedule each station with a power vector that is optimal.
2) Each mesh STA gets a fair share of the resources.

Let $\Gamma(\alpha)$ be an indicator variable such that

$$\Gamma(\alpha) = \begin{cases} 1 & \alpha = 1 \\ 0 & \text{Otherwise} \end{cases} \quad (5)$$

Then according to [12], Eq. (3) can be represented as following

$$F_\alpha(Tx) = \mathfrak{P}_{ij}\left(\Gamma(\alpha)\log(Tx) + (1-\Gamma(\alpha))\frac{Tx^{(1-\alpha)}}{(1-\alpha)}\right) \quad (6)$$

For the sake of notational simplicity let us also consider

$$\mathscr{X}_{ij} = \{Tx_{ij}, P_{ij}\} \qquad \text{and} \qquad \mathscr{X} = \{\mathscr{X}_{i,j} | \forall i, \forall j\} \quad (7)$$

In this case $P_{ij} = \{P_{ij}^{(t)} | \forall t \in \{1,2...T\}\}$ Clearly the objective functions for scheduling and power allocation is dependent upon $\mathscr{X}_{ij}$. Hence two objectives can be represented in terms of $\mathscr{X}_{ij}$ as following.

$$Schedule(\mathscr{X}) = -\sum_{ij}\left(F_\alpha\left(Tx_{ij}\right)\right) \quad (8)$$

$$Power(\mathscr{X}) = \sum_{ij}\sum_{t}\left(P_{ij}^{(t)}\right) \quad (9)$$

As mentioned earlier, Eq. (8) is to ensure $(\mathfrak{P}, \alpha)$-Proportional fairness. Here $\mathfrak{P}_{ij}$ denotes the priority of the sub-flow $i \rightarrow j$ which can be determined by the service class of the flow. However required power vector needs to be minimized as well which can be represented as per Eq. (9). The negative sign in Eq. (8) is used to keep homogeneity of the optimization objective (i.e minimization). Further, each flow must follow the SINR constraint,

$$\Phi[X_{ijh}^{(t)} - 1] - G_{ij}P_{ij}^{(t)} + \gamma(r_h)\sum_{fs} G_{fj}P_{fs}^{(t)} + \gamma(r_h)\eta \leq 0 \quad (10)$$

Here Φ is a substantially large constant which is used to eliminate redundant SINR constraints on sub-flows that are not scheduled at t. Also each mesh STA must be scheduled in such a way that each receiver is different. This is called hidden node constraint. Following equation captures this constraint.

$$\sum_h\left[\sum_{ij}X_{ijh}^{(t)} + \sum_{jf}X_{jfh}^{(t)}\right] \leq 1 \quad (11)$$

Now the entire Vector Optimization can be expressed as following.

Problem 1 (Vector Optimization Problem):

$$\text{Minimize} \quad \mathscr{Q}(\mathscr{X}) = \{Schedule(\mathscr{X}), Power(\mathscr{X})\} \quad (12a)$$

S.T:

$$0 \leq P_{ij}^{(t)} \leq P_{max} \quad h \in \{1,2...m\} \quad t \in \{1,2...DTIM\} \quad (12b)$$

$$\sum_h\left[\sum_{ij}X_{ijh}^{(t)} + \sum_{jf}X_{jfh}^{(t)}\right] \leq 1 \quad (12c)$$

$$\Phi[X_{ijh}^{(t)} - 1] - G_{ij}P_{ij}^{(t)} + \gamma(r_h)\sum_{fs} G_{fj}P_{fs}^{(t)} + \gamma(r_h)\eta \leq 0 \quad (12d)$$

Lemma 1: Every solution of Problem 1 yields a feasible transmission scenario at each time slot.

Proof: Let $\{S(t), P(t)\}$ be a feasible solution of the Problem 1 for any arbitrary scenario and time slot t such that $S(t) = \{i_1 \underset{R(1,1)}{\rightarrow} j_1, i_2 \underset{R(2,2)}{\rightarrow} j_2, ...i_m \underset{R(m,m)}{\rightarrow} j_m\}$ and $P(t) = \{P_{i_1j_1}^{(t)}, P_{i_2j_2}^{(t)}, ...P_{i_mj_m}^{(t)}\}$. Eq. (12c) guarantees that all transmitter and receiver are distinct in each time slot and thus in t also. This prevents hidden node scenario. Let $i_k \underset{R(k,k)}{\rightarrow} j_k$ be any arbitrary transmission in $S(t)$ with power profile $P_{i_kj_k}^{(t)}$. Hence based on Eq. (12d), Eq (4) for SINR constraint gets satisfied. This proves that $\{S(t), P(t)\}$ generates all feasible transmission scenario for all the time slots. ■

Theorem 1: All optimum solutions of Problem 1 generates a Pareto optimal power vector allocation based on the transmissions scheduled in each time slot.

Proof: Let $\{S^*(t), P^*(t)\}$ be an optimum solution for Problem 1 at any arbitrary time slot t such that $S^*(t) = \{i_1 \underset{R(1,1)}{\rightarrow} j_1, i_2 \underset{R(2,2)}{\rightarrow} j_2, ...i_m \underset{R(m,m)}{\rightarrow} j_m\}$ and $P^*(t) = \{P_{i_1j_1}^{*(t)}, P_{i_2j_2}^{*(t)}, ...P_{i_mj_m}^{*(t)}\}$. Let the Pareto optimal solution be $P'(t)$. To prove the Pareto optimality of $P^*(t)$, it is enough to prove that $P^*(t) = P'(t)$, component wise. Since there exists only m simultaneous transmissions at t hence only m instances of Eq. (12d) are non redundant. These active constraints can be expressed as follows

$$G_{i_kj_k}P_{i_kj_k}^{*(t)} - \gamma(r_k)\sum_{x \neq k} G_{i_xj_k}P_{i_xj_x}^{*(t)} \geq \gamma(R(k,k))\eta \quad (13)$$

Now the set of constrains in Eq. (13) has a non-negative solution such that $P^*(t) \geq P'(t)$ component wise. This statement can be directly proved by using Perron-Frobenius theorem as [13].

Again the optimality of $P^*(t)$ suggests the fact that $P^*(t) \leq P'(t)$. So we can conclude $P^*(t) = P'(t)$. ■

It can be shown that Problem 1 is $\mathcal{NP}$-hard. For that, a simplistic form of the original problem is considered which incorporates the assumption of $\alpha = 0$, fixed power and fixed MCS. Also protocol interference model [14] is considered. If each sub-flow requires only one transmission opportunity then maximum independent set problem can be easily reduced to the scheduling problem for each time slot. As maximum independent set is a known $\mathcal{NP}$-Complete problem thus we can say Fair-JPRS problem is also $\mathcal{NP}$-hard.

Lemma 2: $Schedule(\mathscr{X})$ is differentiable under $\mathscr{X}_{uv}$ and a convex function.

Proof: Differentiating Eq. (6) w.r.t Tx,

$$\frac{\partial F_\alpha(Tx)}{\partial Tx} = \mathfrak{P}_{Tx}\left(\frac{\Gamma(\alpha)}{Tx} + \frac{(1-\Gamma(\alpha))}{Tx^\alpha}\right) \tag{14}$$

Similarly from Eq. (7),

$$\frac{\partial f(\mathscr{X})}{\partial Tx_{ij}} = \begin{bmatrix}\frac{\partial f(\mathscr{X})}{\partial Tx_{ij}} & \frac{\partial f(\mathscr{X})}{\partial P_{ij}}\end{bmatrix}$$

From Eq. (6)- Eq. (14),

$$\begin{aligned}\frac{\partial Schedule(\mathscr{X})}{\partial \mathscr{X}_{uv}} &= \begin{bmatrix}\frac{\partial Schedule(\mathscr{X})}{\partial Tx_{uv}} & \frac{\partial Schedule(\mathscr{X})}{\partial P_{uv}}\end{bmatrix}\\ &= \begin{bmatrix}-\mathfrak{p}_{uv}\left(\frac{\Gamma(\alpha)}{Tx_{uv}} + \frac{(1-\Gamma(\alpha))}{Tx_{uv}{}^\alpha}\right) & 0\end{bmatrix}\end{aligned} \tag{15}$$

So the Hessian matrix can be defined as,

$$\begin{aligned}[H_s] &= \frac{\partial^2 Schedule(\mathscr{X})}{\partial \mathscr{X}_{uv}^2}\\ &= \begin{bmatrix}\frac{\partial^2 Schedule(\mathscr{X})}{\partial Tx_{uv}^2} & \frac{\partial^2 Schedule(\mathscr{X})}{\partial P_{uv}\partial Tx_{uv}}\\ \frac{\partial^2 Schedule(\mathscr{X})}{\partial Tx_{uv}\partial P_{uv}} & \frac{\partial^2 Schedule(\mathscr{X})}{\partial P_{uv}^2}\end{bmatrix}\\ &= \begin{bmatrix}\mathfrak{p}_{uv}\left(\frac{\Gamma(\alpha)}{Tx_{uv}^2} + \frac{(1-\Gamma(\alpha))}{Tx_{uv}{}^{-(\alpha+1)}}\right) & 0\\ 0 & 0\end{bmatrix}\end{aligned} \tag{16}$$

$Tx_{uv} > 0$ as it signifies the data transmitted in each DTIM interval for each active flow $u \to v$. $Tx_{uv} = 0$ signifies flow $u \to v$ is inactive for a particular DTIM interval. $[H_s]$ is positive semi-definite as it is symmetric and all its diagonal elements are non negative. Therefore $Schedule(\mathscr{X})$ is a convex function. ∎

Lemma 3: $Power(\mathscr{X})$ is differentiable under $\mathscr{X}_{uv}$ and is a convex function.

Proof:

$$\frac{\partial Power(\mathscr{X})}{\partial \mathscr{X}_{uv}} = \begin{bmatrix}\frac{\partial Power(\mathscr{X})}{\partial Tx_{uv}} & \frac{\partial Power(\mathscr{X})}{\partial P_{uv}}\end{bmatrix} = \begin{bmatrix}0 & 1\end{bmatrix} \tag{17}$$

Hence the Hessian matrix is,

$$\begin{aligned}[H_p] = \frac{\partial^2 Power(\mathscr{X})}{\partial \mathscr{X}_{uv}^2} &= \begin{bmatrix}\frac{\partial^2 Power(\mathscr{X})}{\partial^2 Tx_{uv}} & \frac{\partial^2 Power(\mathscr{X})}{\partial P_{uv}\partial Tx_{uv}}\\ \frac{\partial^2 Power(\mathscr{X})}{\partial Tx_{uv}\partial P_{uv}} & \frac{\partial^2 Power(\mathscr{X})}{\partial^2 P_{uv}}\end{bmatrix}\\ &= \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}\end{aligned} \tag{18}$$

Evidently $[H_p]$ is positive semi-definite. Hence it can be concluded that $Power(\mathscr{X})$ is convex function. ∎

Lemma 4: For a feasible transmission scenario, constraints in Eq. (12d) is differentiable under $\mathscr{X}_{uv}$ and convex.

Proof:

$$\frac{\partial P_{uv}^{(t)}}{\partial P_{uv}} = \frac{1}{\frac{\partial P_{uv}}{\partial P_{uv}^{(t)}}} = 1 \qquad \frac{\partial X_{ijh}^{(t)}}{\partial Tx_{uv}} = \frac{1}{r_h\sigma}$$

Under feasible transmission scenario system of constraints, Eq. (12d) presents two types of non-redundant constraints.

$$\Phi[X_{ijh}^{(t)} - 1] - G_{ij}P_{ij}^{(t)} + \gamma(r_h)\sum_{fs} G_{fj}P_{fs}^{(t)} + \gamma(r_h)\eta \leq 0 \tag{19}$$

$$\Phi[X_{kqh}^{(t)} - 1] - G_{kr}P_{kq}^{(t)} + \gamma(r_h)G_{iq}P_{ij} + \gamma(r_h)\sum_{fs\neq ij} G_{fq}P_{fs}^{(t)} + \gamma(r_h)\eta \leq 0 \tag{20}$$

Let L.H.S of Eq. (19) and Eq. (20) be denoted as $SINR(\mathscr{X})$ and $SINR'(\mathscr{X})$, respectively. Hence,

$$\frac{\partial SINR(\mathscr{X})}{\partial P_{uv}} = \frac{\partial SINR(\mathscr{X})}{\partial P_{uv}^{(t)}}\frac{\partial P_{uv}^{(t)}}{\partial P_{uv}} = -G_{uv}$$

Similarly,

$$\frac{\partial SINR'(\mathscr{X})}{\partial P_{uv}} = \gamma(r_h)G_{uq}, \quad \frac{\partial SINR(\mathscr{X})}{\partial Tx_{uv}} = \frac{\Phi}{r_h\sigma}, \quad \frac{\partial SINR'(\mathscr{X})}{\partial Tx_{uv}} = 0$$

$$\frac{\partial SINR(\mathscr{X})}{\partial \mathscr{X}_{uv}} = \begin{bmatrix}\frac{\partial SINR(\mathscr{X})}{\partial Tx_{uv}} & \frac{\partial SINR(\mathscr{X})}{\partial P_{uv}}\end{bmatrix} = \begin{bmatrix}\frac{\Phi}{r_h\sigma} & -G_{uv}\end{bmatrix} \tag{21}$$

$$\frac{\partial SINR'(\mathscr{X})}{\partial \mathscr{X}_{uv}} = \begin{bmatrix}\frac{\partial SINR'(\mathscr{X})}{\partial Tx_{uv}} & \frac{\partial SINR'(\mathscr{X})}{\partial P_{uv}}\end{bmatrix} \tag{22}$$

$$= \begin{bmatrix}0 & \gamma(r_h)G_{uq}\end{bmatrix} \tag{23}$$

∎

By the use of Lemma 2, Lemma 3 and Lemma 4 it can be said that Problem 1 is itself a multi-objective convex optimization problem. Hence according to [15], $\mathscr{X}^*$ is a Pareto optimum solution of Problem 1 iff Eq. (24)-(26) has non-negative solution $\forall i : \lambda_i$.

$$\lambda_1\frac{\partial Schedule(\mathscr{X}^*)}{\partial \mathscr{X}_{uv}} + \lambda_2\frac{\partial Power(\mathscr{X}^*)}{\partial \mathscr{X}_{uv}} + \lambda_3\frac{\partial SINR(\mathscr{X}^*)}{\partial \mathscr{X}_{uv}} + \sum_q \lambda_4 q\frac{\partial SINR'(\mathscr{X}^*)}{\partial \mathscr{X}_{uv}} = \begin{bmatrix}0 & 0\end{bmatrix} \tag{24}$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \sum_q \lambda_4 q = 1 \tag{25}$$

$$\lambda_i \geq 0 \tag{26}$$

Eq. (15), Eq. (17),Eq. (21),Eq. (22) and Eq. (24) - Eq. (26) reduce to,

$$\lambda_1 \mathfrak{P}_{uv} \left(\frac{\Gamma(\alpha)}{Tx_{uv}} + \frac{1-\Gamma(\alpha)}{Tx_{uv}^{\alpha}} \right) - \lambda_3 \frac{\Phi}{r_h \sigma} = 0 \tag{27}$$

$$\lambda_2 - \lambda_3 G_{uv} + \sum_q \lambda_{4q} \gamma(r_h) G_{uq} = 0 \tag{28}$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \sum_q \lambda_{4q} = 1 \tag{29}$$

For the sake of simplicity consider $\forall q : \lambda_{4q} = \lambda_4'$ and $\sum_q \lambda_{4q} = \lambda_4$ which further simplify Eq. (27)-(29) into the following,

Problem 2:

$$\lambda_1 \mathfrak{P}_{uv} \left(\frac{\Gamma(\alpha)}{Tx_{uv}} + \frac{1-\Gamma(\alpha)}{Tx_{uv}^{\alpha}} \right) = \lambda_3 \frac{\Phi}{r_h \sigma} \tag{30a}$$

$$\lambda_2 + \gamma(r_h) \lambda_4' \sum_q G_{uq} = \lambda_3 G_{uv} \tag{30b}$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1 \tag{30c}$$

In Problem 2, Tx_{uv}, G_{uv} and the rest of the constants are positive. Hence it can be shown that there exists a value assignment for $\lambda_1, \lambda_2, \lambda_3, \lambda_4'$ such that they are non-negative.

According to the discussion above, there exists a Pareto optimal solution for Problem 1. However as mentioned earlier, Problem 1 is $\mathcal{NP}$-hard and by virtue of reduction, so is Problem 2. Therefore, in the next section a distributed heuristic is proposed for Fair-JPRS problem.

V. Exploring the Pareto Optimality: A Distributed Heuristic for Fair-JPRS

Each mesh STA controls the transmit power and MCS by sensing the channel condition. Based on the interference level of the channel, each mesh STA is granted some slots (TXOP). However, the interference level of the receiver is very hard to know for a particular flow at the time of transmission. At the start of the protocol, each station uses a test signal with P_{max} to serve the purpose of channel condition assessment. A mesh STA v stores SINR of the received frame from another mesh neighbor u in a variable $\mathcal{S}_{uv}$. $\mathcal{S}_{uv}$ is measured in dB and is piggybacked with MCCA advertisement frames. Before sending a frame, u can always use this piggybacked information to get an estimate of the interference in v by using the following equation which is derived from Eq. (4).

$$\sum_q G_{qv} = \frac{1}{P_{max}} \left(\frac{G_{uv} P_{max}}{\mathcal{S}_{uv}} - \eta \right) \tag{31}$$

Considering $\lambda_2 = \lambda_4'$, the following equation can be derived from Eq. (30b).

$$\lambda_3 = \lambda_2 \frac{1 + \gamma(r_h) \sum_q G_{uq}}{G_{uv}} \tag{32}$$

Considering $\eta \mathcal{S}_{uv}$ is negligible we get,

$$\frac{1}{\lambda_3} \approx \frac{1}{\lambda_2} + \frac{\gamma(r_h) G_{uv}}{\lambda_2 \mathcal{S}_{uv}} \tag{33}$$

From Eq. (30a), Eq. (34) can be obtained as follows.

$$Tx_{uv} = \begin{cases} \frac{\lambda_1 \mathfrak{P}_{uv} r_h \sigma}{\lambda_3 \Phi} & \alpha = 1 \\ (\frac{\lambda_1 \mathfrak{P}_{uv} r_h \sigma}{\lambda_3 \Phi})^{(1/\alpha)} & \text{Otherwise} \end{cases} \tag{34}$$

Eq. (31), Eq. (32) and Eq.(34) are used for solving Problem 2.

Each mesh STA v transmits a periodic beacon frame using P_{max} at the start of the DTIM interval. SINR of the received beacon by mesh STA u is denoted by $\mathcal{S}_{uv}$. Each mesh STA broadcasts its $\mathcal{S}_{uv}$ with MCCAOP advertisement request message. This requires additional frame overhead of 2 Bytes. A mesh STA with highest $\mathcal{S}_{uv}$ in it's neighborhood is termed as "winner", and prioritized to reserve the channel. The assigned rate r_h for winner is chosen so that it is highest achievable data rate with receive sensitivity less than or equal to $\mathcal{S}_{uv}$ *i.e.* $\gamma(r_h) \leq \mathcal{S}_{uv} < \gamma(r_{h+1})$.

$$\mathcal{S}_{uv} = \frac{G_{uv} P_{max}}{\mathcal{I}} \tag{35}$$

$$\gamma(r_h) \leq \frac{G_{uv} P_{uv}^{(h)}}{\mathcal{I}} \tag{36}$$

Here $\mathcal{I}$ represents the cumulative interference and ambient noise. Using Eq. (35) and Eq. (36), we can derive,

$$P_{uv}^{(h)} \geq \frac{\gamma(r_h)\, P_{max}}{\mathcal{S}_{uv}} \tag{37}$$

Though for theoretical modeling purpose this work assumes that each mesh STA has $P_{ij}^{(t)} \in [0, P_{max}]$, in practice commodity routers come with fixed discrete power levels. In that particular case, $P_{uv}^{(h)}$ is approximated to the next higher power level available in routers power level set.

Based on the solution of Problem 2, which is a simplified version of Problem 1, MCCAOP owner calculates MCCAOP parameters in the following way.

1) MCCAOP offset is set as the first free slot in the MCCAOP owner's reservation set.
2) MCCAOP Periodicity is used for ensuring short term fairness of the transmissions. To provide each mesh STA with an equal chance to transmit, this value is set as the number of the MCCA enabled mesh neighbors(Δ).
3) If the winner does not have any prior schedule information, it assigns MCCAOP duration with a predefined fixed value Tx_{max}. If the winner already has a schedule, it estimates $\overline{\lambda}$ using Eq. (31)- (33). Based on the estimated $\overline{\lambda}$, it solves Problem 2. MCCAOP duration is set based on calculated $\lceil \frac{Tx_{uv}}{\Delta} \rceil$.

As per standard, MCCAOP parameters, along with MAF of the MCCA owner, are suggested to the MCCAOP responder.

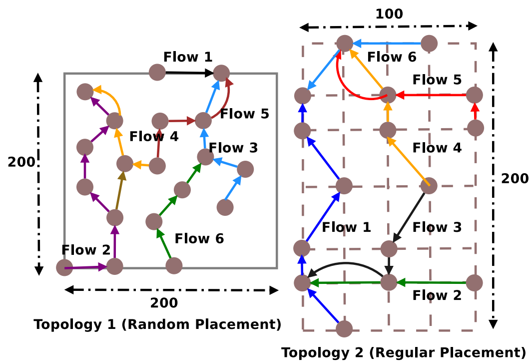

Fig. 1: Simulation Topologies

A MCCAOP responder accepts the schedule if it satisfies the following conditions.

1) MAF is less than MAF limit.
2) All transmission opportunity slots, (TXOP)s do not overlap with the interference period of the responder.
3) All TXOPs must not conflict with the neighborhood MCCAOP periods of the MCCAOP owner.

If the last two conditions are not satisfied, the responder recalculates TX_{uv} from the received MCCAOP parameters and re-computes MCCAOP parameters for an alternate solution. This alternate MCCAOP parameters are sent back to the MCCAOP owner via MCCAOP Setup reply frame with proper reject code. After receiving successful reservation reply from all MCCAOP responders, MCCAOP owner broadcasts MCCAOP advertisement frame to let the mesh neighbors know about the successful reservation. Once the MCCASCANDURATION is over, each mesh STA starts transmission based on the accepted schedule, data rate and transmit power.

VI. Performance Analysis and Comparison

For determining the performance of the proposed scheme, simulation experiments are performed for two different topologies given in Fig. 1. The different flows are color coded in the two topologies. Simulations are performed in NS-3.18 [16]. Each mesh STA is equipped with single omni-directional antenna. 802.11n physical layer and 802.11s MAC with SDR support are considered for each mesh STA. A single channel is used for the entire duration of the simulation. Channel is time divided into 0.8ms slots. Initial 40 slots are reserved for control and Beacon frames. Rest 1000 slots are used for actual data communication (DTIM). Control frames are transmitted using maximum transmit power and minimum data rate available. For solving Problem 2, α has been chosen uniformly between the range of 3 to 8. More experimental results, performed to realize the effect of α, are omitted due to the space constraint of the paper. Rest of the simulation parameters are given in the Table I. The mesh STAs communicate with the mesh gates through multi-hop communication. Static routing protocol is used for frame forwarding to minimize the influence of routing in each scenario. Each simulation experiment is executed for at-least 10 times. The proposed protocol is compared with the standard MCCA and DPRL [2].

Frame Size	512 B	
Traffic Generation rate	15Mb/s	
MCS	Data Rate	Receive Sensitivity
6.5OFDM	6.5Mbps	-87dBm
26OFDM	26Mbps	-81dBm
39OFDM	39Mbps	-78dBm
54OFDM	54Mbps	-73dBm
Min Power Level	2dbm	
Max Power Level	17dbm	
Power Levels	9	
Slot Time σ	0.80ms	
DTIM	1s	
Slots/DTIM	1000	
Scan Duration	32ms	

TABLE I: Simulation Parameters

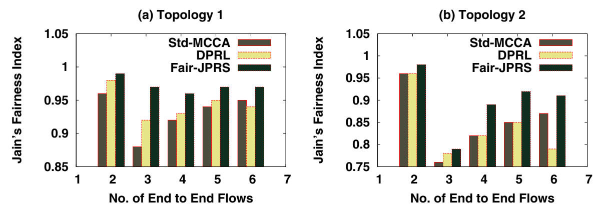

Fig. 2: Effect on Jain's Fairness Index

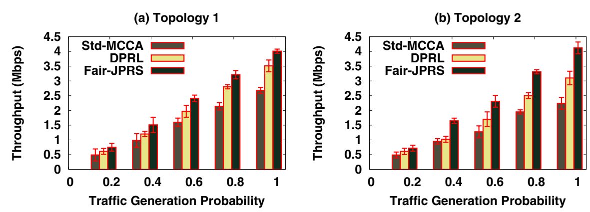

Fig. 3: Effect on End To End Throughput

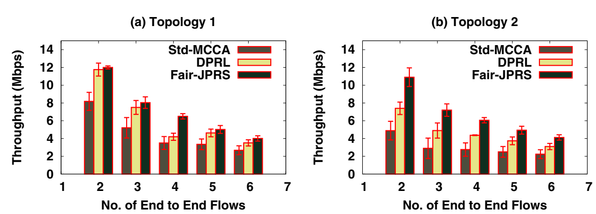

Fig. 4: Effect on End To End Throughput

Simulation results in Fig. 2 show the variation of Jain's fairness index [17], with respect to the number of active end to end flows. In all of the cases, Fair-JPRS gives a better result compared to DPRL and standard MCCA. Fair-JPRS considers neighbor coordinations during transmit power adaptation that helps in improving per node fairness. However in Fig. 2 (b), the graph shows a sharp change for all the protocols when only three flows are active. The reason behind this is, Flow 1 and Flow 3 has a shared bottleneck link and due to this bottleneck link the throughput of the corresponding flow reduces. However Flow 2 does not have any such link. Therefore due to the effect of the specific topology and flow distribution, all of the protocols show lower fairness index.

The improvement in fairness results in significant performance boost in the average end to end throughput. This effect

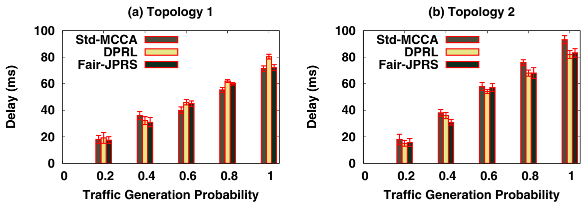

Fig. 5: Effect on End To End Delay

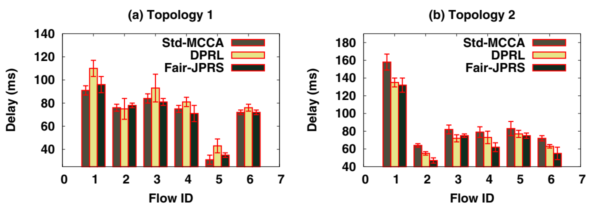

Fig. 6: Effect on End To End Delay

has been captured in Fig. 3[2] and Fig. 4. The confidence factor for average end to end throughput has been calculated based on the standard deviation of all active flows. The nature of the graphs suggest that Fair-JPRS outperforms the rest of two protocols. Per node fairness has direct impact over the end-to-end throughput performance by avoiding the flow stalling at intermediate mesh STAs. As a consequence effect of fairness improvement, Fair-JPRS provides better end-to-end throughput compared to other two schemes. The graphs reveal that depending on the topology and flow distribution scenarios, Fair-JPRS can result in a $10\% - 40\%$ improvement in the end-to-end throughput, compared to DPRL.

It has been studied by Gamal *et.al.* [18] that improvement in throughput might result in the increase in forwarding delay. To analyze the effect of forwarding, we present the variation in delay for all of the protocols. Fig. 5 captures the variation of average delay in case of different data generation probabilities. When all the flows are scheduled, end to end delay of each flow is calculated and shown in Fig. 6. In case of Topology 2, Flow 2 and Flow 3 suffer extra delay than that of DPRL because it receives less bandwidth to ensure fairness for other contending flows. However the difference is very small and for most of the cases the delay remains less than or equal to the end to end delay for DPRL.

VII. Conclusion

Network performance in terms of throughput optimization is a challenge for WMN. In this paper, a fair joint transmit power and rate scheduling problem has been studied in the context of IEEE 802.11s wireless mesh network. For ensuring effective power allocation and throughput fairness, a vector optimization problem is formulated. It has been proved that the problem is a convex optimization problem. The existence of Pareto optimal solution of the problem is proved in this work. However the problem is a $\mathcal{NP}$-hard problem. Therefore a heuristic is also proposed for solving the problem by simplifying the assumptions of the centralized problem. The heuristic proposal is augmented with the IEEE 802.11s standard MCCA. Finally the performance of the scheme is evaluated with the help of simulation results. The simulation reveals that the augmented scheme performs well in terms of end to end throughput. The fairness of proposed protocol shows significant improvement over existing works.

[2]Note: Red marker in the plot, indicates standard deviation

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